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Theoretical evaluations of the running alpha and the muon magnetic moment

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Abstract. The status of the theoretical predictions for the anomalous magnetic moment of the muon, $g-2$, and for the electromagnetic coupling α at the scale M_Z are reviewed. We discuss recent developments and present new evaluations, which take into account re-analysed data from CMD-2 in Novosibirsk.

PACS. 13.40.Em Electric and magnetic moments – 12.38.Lg Other nonperturbative calculations – 14.60.Ef Muons – 13.66.Bc Hadron production in e-e+ interactions

1 $g-2$ of the muon

The magnetic moment of the muon, $\mu = -g\, e\, \mathbf{s}/(2m)$ (where e and m are the particle's charge and mass), is a fundamental observable in elementary particle physics. The Dirac equation gives $g = 2$, but quantum corrections lead to the anomalous magnetic moment, $a_\mu \equiv (g-2)/2$. The largest contribution to a_μ comes from QED, but also electroweak (EW) and hadronic corrections contribute: $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$. Experimentally, a_μ is one of the most precisely measured quantities. The current world avarage is $a_\mu = (11\ 659\ 203 \pm 8) \cdot 10^{-10}$ and comes from the E821 experiment at BNL [1]. With this outstanding precision, $g-2$ constitutes a very strong test of all sectors of the Standard Model (SM) and could eventually, if a significant discrepancy between experimental and theoretical value can be established, signal physics beyond the SM.

In Table 1 we present the different contributions to a_μ in the SM. The electromagnetic and EW contributions are known to very high accuracy. However, as is clear from Table 1, the hadronic contributions are not known to comparable accuracy. They are usually divided into leading and next-to-leading order contributions from vacuum polarization induced corrections, and the so-called light-by-light scattering contributions: $a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,NLO} + a_\mu^{\rm had,L-by-L}$. These hadronic contributions cannot be calculated in perturbative QCD (pQCD). For the L-by-L part model dependent estimates have to be used, whereas for the vacuum-polarization induced corrections dispersion relations exist which allow to calculate these hadronic contributions using experimental data, $\sigma^0_{\rm had}(s)$.

For the leading order the dispersion relation reads

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} {\rm d}s\, \sigma^0_{\rm had}(s) K(s), \qquad (1)$$

where $K(s) = m_\mu^2/(3s)\cdot(0.63\ldots1)$ is a monotonic kernel function. In (1) low energies are weighted much stronger than higher energies. This behaviour is also visible in Table 2, where we list the contributions to $a_\mu^{\rm had,LO}$ from different energy regions. The numbers here are from the group HMNT [2]. For their most recent analysis and a complete list of refences see [3], for other recent analyses see e.g. [4,5]. All these analyses have in common that they are based on data, and do not attempt to use pQCD in the low energy regime below the bottom threshold. Differences occur in the choice, treatment and combination of data, see [3,4,5] and references therein for further details. However, three issues shall be briefly discussed here:

(i) In the analysis of HMNT an ambiguity in the data input is found in the region between $\sqrt{s} = 1.4\ldots2$ GeV. There one has the choice to either sum up the measurements of the many available exclusive hadronic channels or to rely on the available lowest energy inclusive measurements for $\sigma^0_{\rm had}(s)$. This ambiguity is displayed in the third and forth line and the last two lines of Table 2. HMNT perform a QCD sum-rule analysis which strongly favours the inclusive data.

(ii) DEHZ analyse and use in addition τ spectral function data and find those larger and incompatible with e^+e^- data. The use of τ data relies on the conserved vector current hypothesis (Isospin-symmetry), and complicated corrections for Isospin-breaking effects have to be applied. Fig. 1 displays $e^+e^- \to \pi^+\pi^-$ data compared to a compilation of the τ data (left plot) as used by DEHZ. The plot on the right is a zoom into the $\rho-\omega$ interference regime in the $\pi^+\pi^-$ channel and shows data (as points) and their combination (as a band) as used by HMNT. It is

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Table 1. The different contributions to a_μ in the Standard Model

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\ 584\ 703.5 \pm 2.8$	up to 5-loop! (Laporta+Remiddi, Kinoshita *et al.*)
	(was $116\ 584\ 705.7 \pm 2.9$)	→ incl. recent correction from Kinoshita+Nio
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		→ agrees with Knecht+Peris+Perrottet+de Rafael
LO hadr.	$7090 \pm 51 \pm 12 \pm 28$	Davier+Eidelman+Hoecker+Zhang '03a (τ)
	$6847 \pm 60 \pm 36$	Davier+Eidelman+Hoecker+Zhang '03a (e^+e^-)
	$6918 \pm 58 \pm 20$	HMNT, as presented at EPS03, w. CMD-2 re-anal.
NLO hadr.	-100 ± 6	Alemany *et al.* '98 in agreem. with Krause '97
L-by-L	80 ± 40	compilation from Nyffeler, `hep-ph/0203243`
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$(11659175.6 \pm 7.4)10^{-10}$	with HMNT (using e^+e^-)
Exp.:	$(11\ 659\ 203 \pm 8) \cdot 10^{-10}$	BNL E821 world average 2002
$a_\mu^{EXP} - a_\mu^{TH}$	$(27.4 \pm 11) \cdot 10^{-10}$	$2.5\,\sigma$ (HMNT, e^+e^-), $0.9\,\sigma$ using τ (DEHZ)

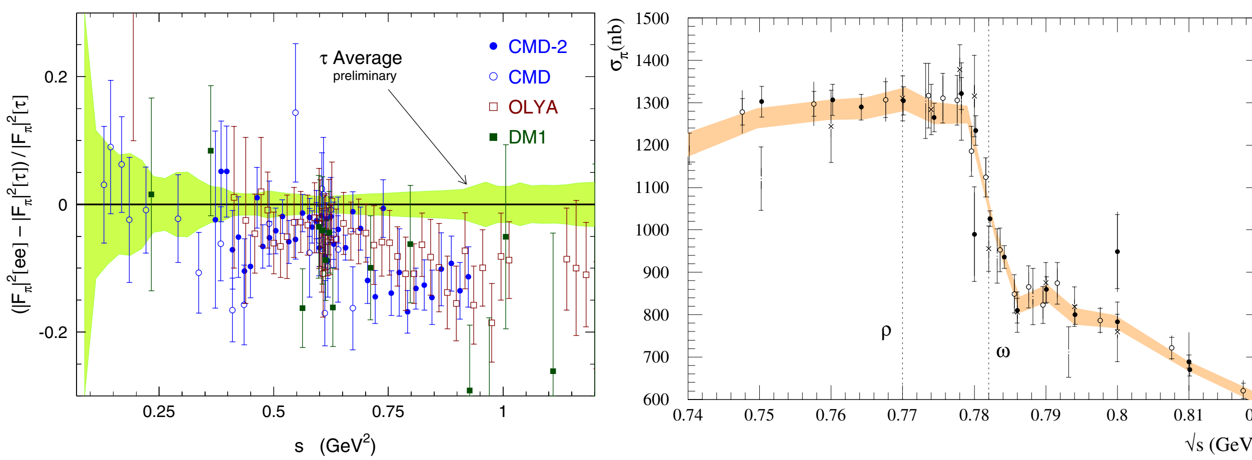

Fig. 1. Averaged τ data (including corrections) compared to the $e^+e^- \to \pi^+\pi^-$ data (*left plot*, figure from DEHZ [4]) and zoom into the $\rho - \omega$ interference region of the $\pi^+\pi^-$ channel (*right plot*, data and fit as used by HMNT [2,3])

at present not clear where the discrepancy between e^+e^- and τ data comes from. It may be in the data or in the corrections applied to them (for a discussion see [6]). However, preliminary results from KLOE, using the method of radiative return, seem to confirm the e^+e^- data [7,8].

(iii) All analyses rely strongly on the most precise data from CMD-2 for the dominant $\pi^+\pi^-$ channel. These data have recently been re-analysed and corrected upwards [9], which in turn shifts the predictions for $a_\mu^{\rm had}$ to higher values. At the time of the conference HMNT had presented first such results with preliminary re-analysed data from CMD-2. For the numbers presented here we have used their published results [9].

In Fig. 2 we finally display recent predictions for $g-2$ of the muon compared to the experimental value from BNL. The theoretical (e^+e^- based) predictions agree well with each other, and the theoretical and experimental accuracy are comparable. However, at present there persists a discrepancy of about $2\,\sigma$ (if one relies on e^+e^- data input only), even after the most recent shift due to the re-analysis of the CMD-2 data.

Table 2. Contributions to $a_\mu^{\rm had,LO}$ from different energy regions (energies in GeV)

energy range	comments	$a_\mu^{\rm had,\ LO} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.36 ± 0.05
$0.32 \ldots 1.43$	excl. only	605.39 ± 5.15
$1.43 \ldots 2.00$	excl. only	35.98 ± 1.68
	incl. only	32.41 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.12 ± 1.14
$J/\psi + \psi(2S)$	NW	7.30 ± 0.43
$\Upsilon(1-6S)$	NW	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.09 ± 0.01
$\sum$ of all	'excl.'	695.33 ± 5.61
	'incl.'	691.77 ± 5.84

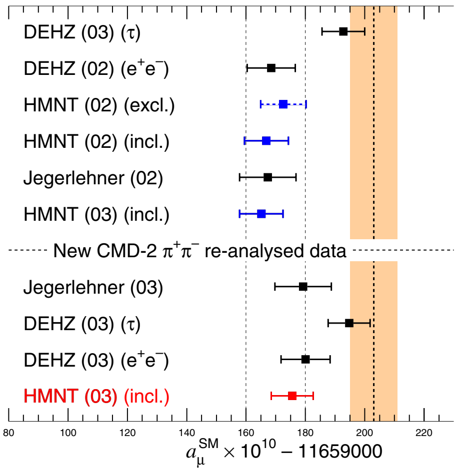

Fig. 2. Recent theoretical evaluations of a_μ in the Standard Model compared to the BNL 02 world average

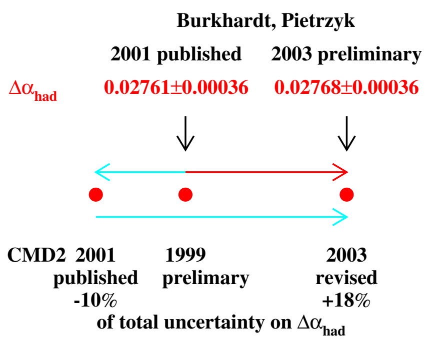

Fig. 3. Impact of the CMD-2 re-analysis on $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$

2 $\alpha(M_Z)$

With our comprehensive data compilation for $R = \sigma^0_{\rm had}(s)/(4\pi\alpha^2/(3s))$ we can also calculate the hadronic contributions to the electromagnetic coupling at the scale M_Z, the least well known parameter of (G_μ, M_Z and $\alpha(M_Z^2)$) which define the electroweak theory. We use the same input as in the analysis of a_μ for the dispersion integral, which reads in this case

$$\Delta\alpha_{\rm had}^{(5)} = -\frac{\alpha s}{3\pi}\,{\rm P}\int_{s_{\rm th}}^{\infty} \frac{R(s')\,{\rm d}s'}{s'(s'-s)}. \qquad (2)$$

Here we present the (preliminary) result

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.02785 \pm 0.00022\,, \qquad (3)$$

where the error includes additional contributions from the treatment of radiative corrections. Together with the leptonic (calculated up to three loop accuracy) contribution, $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$, and the top quark contribution, $\Delta\alpha^{\rm top}(M_Z^2) = -0.000076$, we obtain

$$\alpha(M_Z^2)^{-1} = 128.914 \pm 0.029. \qquad (4)$$

In contrast to $g-2$, for $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ the impact of the re-analysis of the CMD-2 data is less important, as the

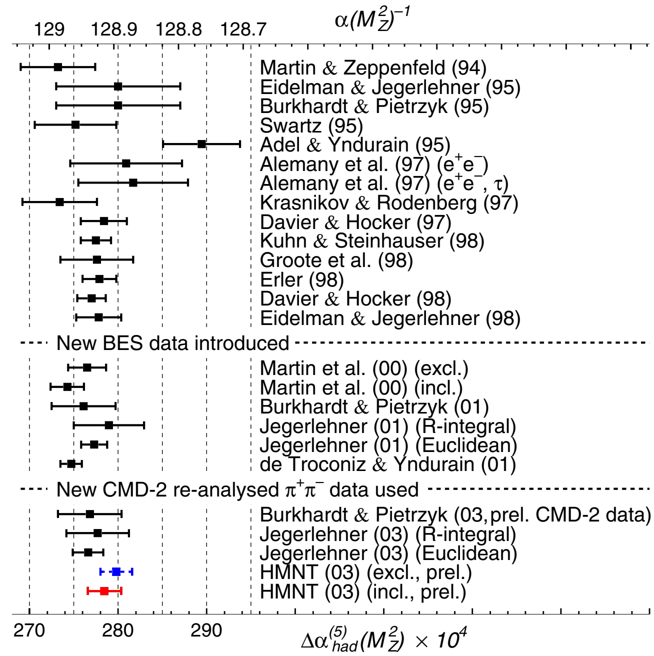

Fig. 4. Comparison of different evaluations of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$

dispersion integral (2) does not give such strong weight to low energies.

The effects, together with updated numbers for the analysis [10], are displayed in Fig. 3. In Fig. 4 we finally display (3) and other evaluations of $\Delta\alpha_{\rm had}^{(5)}(M_Z)$. Our result compares well with other works, but has smaller errors than most other data driven analyses. It seems that not much can be gained by using pQCD in a wider energy range after recent improvements of the data.

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References

1. Muon $(g-2)$ Collaboration, G.W. Bennett et al.: Phys. Rev. Lett. **89**, 101804 (2002), [Erratum-ibid **89**, 129903 (2002)]
2. K. Hagiwara, A.D. Martin, Daisuke Nomura, and T. Teubner: Phys. Lett. B **557**, 69 (2003)
3. K. Hagiwara, A.D. Martin, Daisuke Nomura, and T. Teubner: `arXiv:hep-ph/0312250`
4. M. Davier, S. Eidelman, A. Höcker, and Z. Zhang: `arXiv:hep-ph/0308213`
5. F. Jegerlehner: `arXiv:hep-ph/0310234`
6. S. Ghozzi and F. Jegerlehner: `arXiv:hep-ph/0310181`
7. M. Incagli: in these proceedings
8. J.H. Kühn: in these proceedings
9. CMD-2 Collaboration, R.R. Akhmetsin et al.: `arXiv:hep-ex/0308008`
10. H. Burkhardt and B. Pietrzyk: Phys. Lett. B **513**, 46 (2001)